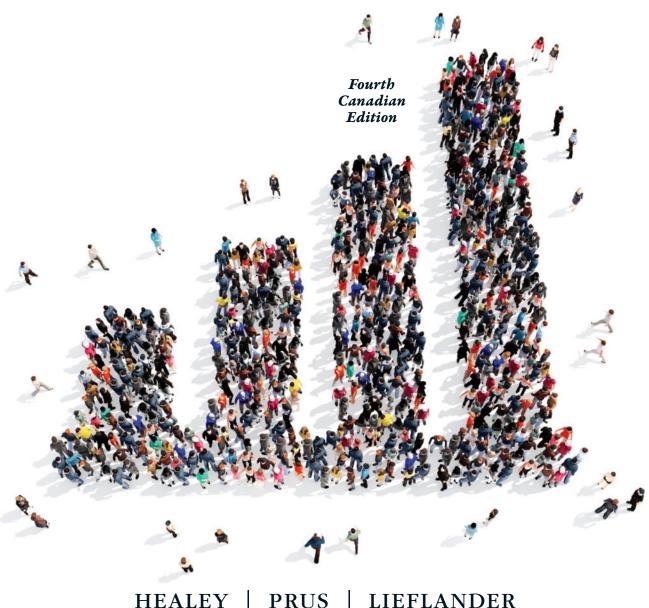
# Statistics

A TOOL FOR SOCIAL RESEARCH



HEALEY | PRUS | LIEFLANDER

#### **FREQUENTLY USED FORMULAS**

#### **CHAPTER 2**

Proportion 
$$p = \frac{f}{n}$$

Percentage 
$$\% = \left(\frac{f}{n}\right) \times 100$$

#### **CHAPTER 3**

Mean (sample) 
$$\overline{X} = \frac{\Sigma(X_i)}{n}$$

Standard deviation (sample)

$$s = \sqrt{\frac{\sum (X_i - \overline{X})^2}{n}}$$

#### **CHAPTER 4**

**Z** scores 
$$Z = \frac{X_i - \overline{X}}{s}$$

#### **CHAPTER 6**

Confidence interval for a sample mean  $(\sigma \text{ known})$ 

c.i. = 
$$\overline{X} \pm Z \left( \frac{\sigma}{\sqrt{n}} \right)$$

Confidence interval for a sample mean  $(\sigma \text{ unknown})$ 

c.i. = 
$$\overline{X} \pm t \left( \frac{s}{\sqrt{n-1}} \right)$$

Confidence interval for a sample proportion

c.i. = 
$$P_s \pm Z \sqrt{\frac{P_u(1-P_u)}{n}}$$

#### **CHAPTER 7**

Chi square 
$$\chi^2$$
 (obtained) =  $\sum \frac{(f_o - f_e)^2}{f_e}$ 

#### **CHAPTER 8**

Phi 
$$\phi = \sqrt{\frac{\chi^2}{n}}$$

Cramer's V

$$V = \sqrt{\frac{\chi^2}{(n)(\text{minimum of } r - 1, c - 1)}}$$

Lambda 
$$\lambda = \frac{E_1 - E_2}{E_1}$$

#### **CHAPTER 9**

Gamma 
$$G = \frac{n_s - n_d}{n_s + n_d}$$

Spearman's rho 
$$r_s = 1 - \frac{6\Sigma D^2}{n(n^2 - 1)}$$

#### **CHAPTER 10**

Means (
$$\sigma$$
 known)  $Z$  (obtained) =  $\frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$ 

Means ( $\sigma$  unknown)

$$t \text{ (obtained)} = \frac{\overline{X} - \mu}{s/\sqrt{n-1}}$$

**Proportions** 
$$Z ext{ (obtained)} = \frac{P_S - P_u}{\sqrt{P_u(1 - P_u)/n}}$$

#### **CHAPTER 11**

Means (
$$\sigma$$
 known)  $Z$  (obtained) =  $\frac{(\overline{X}_1 - \overline{X}_2)}{\sigma_{\overline{x} - \overline{x}}}$ 

Means ( $\sigma$  unknown)

$$t ext{ (obtained)} = \frac{\overline{X}_1 - \overline{X}_2}{\sigma_{\overline{X} - \overline{X}}}$$

Standard deviation of the sampling distribution for sample means ( $\sigma$  known)

$$\sigma_{\overline{x}-\overline{x}} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Standard deviation of the sampling distribution for sample means ( $\sigma$  unknown)

$$\sigma_{\bar{x}-\bar{x}} = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}} \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

Pooled estimate of population proportion

$$P_u = \frac{n_1 P_{s1} + n_2 P_{s2}}{n_1 + n_2}$$

Standard deviation of the sampling distribution for sample proportions

$$\sigma_{p-p} = \sqrt{P_u(1-P_u)} \sqrt{(n_1+n_2)/n_1n_2}$$

**Proportions** 

$$Z \text{ (obtained)} = \frac{(P_{s1} - P_{s2})}{\sigma_{p-p}}$$

#### **CHAPTER 12**

Total sum of squares

$$SST = \sum (X_i - \overline{X})^2$$

Sum of squares between

$$SSB = \sum n_b (\overline{X}_b - \overline{X})^2$$

Sum of squares within

$$SSW = \sum (X_i - \overline{X}_b)^2$$

Degrees of freedom for SSW

$$dfw = n - k$$

Degrees of freedom for SSB

$$dfb = k - 1$$

Mean square within

$$MSW = \frac{SSW}{dfw}$$

Mean square between

$$MSB = \frac{SSB}{dfb}$$

F ratio

$$F = \frac{\text{MSB}}{\text{MSW}}$$

#### **CHAPTER 13**

Least-squares regression line

$$Y = a + bX$$

Slope

$$b = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{n\Sigma X^2 - (\Sigma X)^2}$$

Y intercept

$$a = \overline{Y} - b\overline{X}$$

Pearson's r

$$r = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{[n\Sigma X^2 - (\Sigma X)^2][n\Sigma Y^2 - (\Sigma Y)^2]}}$$

#### **CHAPTER 14**

Partial correlation coefficient

$$r_{yx,z} = \frac{r_{yx} - (r_{yz})(r_{xz})}{\sqrt{1 - r_{yz}^2} \sqrt{1 - r_{xz}^2}}$$

Least-squares multiple regression line

$$Y = a + b_1 X_1 + b_2 X_2$$

Partial slope for X<sub>1</sub>

$$b_1 = \left(\frac{s_y}{s_1}\right) \left(\frac{r_{y1} - r_{y2}r_{12}}{1 - r_{12}^2}\right)$$

Partial slope for X<sub>2</sub>

$$b_2 = \left(\frac{s_y}{s_2}\right) \left(\frac{r_{y2} - r_{y1}r_{12}}{1 - r_{12}^2}\right)$$

Y intercept

$$a = \overline{Y} - b_1 \overline{X}_1 - b_2 \overline{X}_2$$

Beta-weight for X<sub>1</sub>

$$b_1^* = b_1 \left( \frac{s_1}{s_v} \right)$$

Beta-weight for X<sub>2</sub>

$$b_2^* = b_2 \left( \frac{s_2}{s_y} \right)$$

Standardized least-squares regression line

$$Z_{v} = b_{1}^{*} Z_{1} + b_{2}^{*} Z_{2}$$

Coefficient of multiple determination

$$R^2 = r_{y1}^2 + r_{y2.1}^2 (1 - r_{y1}^2)$$

# Statistics A TOOL FOR SOCIAL RESEARCH

Fourth Canadian Edition

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Carleton University

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#### **NELSON**

Statistics: A Tool for Social Research, Fourth Canadian Edition

by Joseph F. Healey, Steven G. Prus, and Riva Lieflander

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Publisher, Digital and Print Content: Leanna MacLean

Marketing Manager: Claire Varley

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Photo and Permissions Researcher: Natalie Barrington **Production Project Manager:** 

Fiona Drego

**Production Service:** MPS Ltd.

Copy Editor: Wendy Yano

Proofreader: Shilbhadra Maity

Indexer: May Hasso

Design Director: Ken Phipps **Higher Education Design PM:** 

Pamela Johnston

Interior Design: Colleen Nicholson

Cover Design: Colleen Nicholson

Cover Image: tai11/Shutterstock

Compositor: MPS Ltd.

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Adapted from Statistics: A Tool for Social Research, 10th Edition, by Joseph F. Healey, published by Cengage Learning. Copyright ©2015 by Cengage Learning.

Printed and bound in the United States

1 2 3 4 21 20 19 18

For more information contact Nelson Education Ltd., 1120 Birchmount Road, Toronto, Ontario, M1K 5G4. Or you can visit our Internet site at nelson.com

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Library and Archives Canada Cataloguing in Publication Data

Healey, Joseph F., 1945-, author Statistics: a tool for social research / Joseph F. Healey, Christopher Newport University, Steven G. Prus, Carleton University, Riva Lieflander, Carleton University. — Fourth Canadian edition.

Includes index.
Issued in print and electronic formats.
ISBN 978-0-17-672508-2 (softcover).
—ISBN 978-0-17-685378-5 (PDF)

1. Social sciences—Statistical methods—Textbooks.
2. Statistics—Textbooks.
3. Textbooks. I. Prus, Steven G. (Steven Gerald), 1967-, author III. Lieflander, Riva, 1966-, author III. Title.

HA29.H42 2018 519.5 C2018-901700-7 C2018-901701-5

ISBN-13: 978-0-17-672508-2 ISBN-10: 0-17-672508-3

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Introduction to Probability

Other EPSEM Sampling Techniques, Systematic, Stratified, and Cluster Samples Nonparametric Hypothesis Testing for Variables Measured at the Ordinal Level

Linear Regression with Dummy Variables

Regression with a Dichotomous Dependent Variable: an Introduction to Logistic Regression

**Elaborating Bivariate Tables** 

### Preface to the Fourth Canadian Edition

Sociology and the other social sciences, including political science, social work, public administration, criminology, and gerontology, are research-based disciplines, and statistics are part of their everyday language. To join the conversation, you must learn the vocabulary of research, data analysis, and scientific thinking. Knowledge of statistics will enable you to understand the professional research literature of your discipline as well as the research reports you encounter in everyday life. Knowledge of statistics will enable you to conduct quantitative research and to contribute to the growing body of social science knowledge.

Although essential, learning statistics can be a challenge. Students in statistics courses typically have a wide range of mathematical backgrounds and an equally diverse set of career goals. They are often puzzled about the relevance of statistics for them, and, not infrequently, there is some math anxiety to deal with.

This textbook introduces statistical analysis for the social sciences while addressing these challenges. The textbook makes minimal assumptions about mathematical background (the ability to read a simple formula is sufficient preparation for virtually all of the material in the textbook), and a variety of special features help students analyze data successfully. The textbook is sufficiently flexible to be used in any program with a social science base.

The textbook is written at a level intermediate between a strictly mathematical approach and a mere "cookbook." We have not sacrificed comprehensive coverage or statistical correctness, but theoretical and mathematical explanations are kept at an elementary level, as is appropriate in a first exposure to social statistics. For example, we do not treat formal probability theory per se. Rather, the background necessary for an understanding of inferential statistics is introduced, informally and intuitively, in Chapters 4 and 5 while considering the concepts of the normal curve and the sampling distribution. The textbook makes no claim that statistics are "fun" or that the material can be mastered without considerable effort. At the same time, students are not overwhelmed with abstract proofs, formula derivations, and mathematical theory, which can needlessly frustrate the learning experience at this level.

GOALS AND FEATURES OF THE TEXTBOOK

The textbook has three primary goals, which are collectively designed to help students to develop the skills necessary to become statistically literate. The first goal is to develop an appreciation for the role of statistics in the research process. The second is to cultivate a competence to perform basic

statistical calculations. The third is to develop a capability to critically read statistical information, especially as reported in the professional research literature. The following discussion considers each goal in more detail and how it is achieved through the many special features of the textbook.

- **1.** An Appreciation of Statistics. To appreciate statistics is to understand the relevance of statistics for research, to analyze and interpret the meaning of a statistical test, and to select and use an appropriate statistic for a given purpose and a given set of variables. This textbook develops these qualities, within the framework of an introductory course, in the following ways:
- The relevance of statistics. The textbook stresses the importance of statistics as a means of analyzing and manipulating data to answer research questions. This central theme of relevance is reinforced by a series of boxes labelled "Applying Statistics," each of which illustrates some specific way in which statistics can be used to answer questions.

Furthermore, the end-of-chapter problems are labelled by the social science discipline or subdiscipline from which they are drawn: SOC for sociology, SW for social work, PS for political science, CI for criminology and criminal justice, PA for public administration, and GER for gerontology. By identifying problems with specific disciplines, students can more easily see the relevance of statistics to their own academic interests. (Not incidentally, they will also see that the disciplines have a large subject matter in common.)

- Interpreting statistics. For most students, interpretation—saying what statistics mean—is a big challenge. The ability to interpret statistics can be developed only by exposure and experience. To provide exposure, we have been careful, in the example problems, to express the meaning of the statistic in terms of the original research question. To provide experience, the end-of-chapter problems almost always call for an interpretation of the statistic calculated. To provide examples, many of the answers to odd-numbered computational problems in the back of the textbook are expressed in words as well as numbers. The "Interpreting Statistics" sections provide additional, detailed examples of how to express the meaning of statistics.
- Using statistics: Ideas for research projects. Appendix E offers ideas for independent data-analysis projects for students. The projects require students to use a computerized statistical package to analyze a data set. They can be assigned at intervals throughout the semester or at the end of the course. Each project provides an opportunity for students to practise and apply their statistical skills and, above all, to exercise their ability to understand and interpret the meaning of the statistics they produce.

- **2. Computational Competence.** Students should emerge from their first course in statistics with the ability to perform elementary forms of data analysis—to execute a series of calculations and arrive at the correct answer. To be sure, computers and calculators have made computation less of an issue today. Yet computation and statistics are inseparable, and since social science majors frequently do not have strong quantitative backgrounds, we have included a number of features to help students cope with these challenges:
- Step-by-step computational algorithms are provided for each statistic.
- Extensive problem sets are provided at the end of each chapter. Most of these problems use fictitious data and are designed for ease of computation.
- Cumulative exercises are included at the end of each part to provide practice in choosing, computing, and analyzing statistics. These exercises present only data sets and research questions. Students must choose appropriate statistics as part of the exercise. The Cumulative exercises can be found at nelson.com/students.
- Solutions to odd-numbered computational problems are provided so that students may check their answers.
- **3. Critically Read Statistical Information.** A statistically literate person can comprehend and critically evaluate statistical information reported in research written by others. The development of this quality is a particular problem at the introductory level since (1) the vocabulary of professional researchers is so much more concise than the language of the textbook, and (2) the statistics featured in the literature are generally more advanced than those covered at the introductory level. To help bridge this gap, we have included a series of boxes labelled "Reading Statistics," beginning in Chapter 1. In each box, we briefly describe the reporting style typically used for the statistic in question and try to alert students about what to expect when they approach the professional literature. These inserts include excerpts from the Canadian research literature and illustrate how statistics are actually applied and interpreted by social scientists.

**Additional Features.** A number of other features make the textbook more meaningful for students and more useful for instructors as they help develop students' statistical literacy:

 Readability and clarity. The writing style is informal and accessible to students without ignoring the traditional vocabulary of statistics. Problems and examples have been written to maximize student interest and to focus on issues of concern and significance. For the more difficult material (such as hypothesis testing), students are first walked through

- an example problem before being confronted by formal terminology and concepts. Each chapter ends with a summary of major points and formulas and a glossary of important concepts. A list of frequently used formulas inside the front cover and a glossary of symbols inside the back cover can be used for quick reference.
- Organization and coverage. The textbook is divided into four parts. Overview sections have been included with each of the four parts of the textbook. These provide a "road map" of the material that follows and help students see why the chapters are organized as they are. Most of the coverage is devoted to the first three parts of the textbook: descriptive statistics, estimation, and hypothesis testing. Estimation and hypothesis testing are the two main applications of inferential statistics. The distinction between description and inference is introduced in Chapter 1 and maintained throughout the textbook. In selecting statistics for inclusion, we have tried to strike a balance between the essential concepts with which students must be familiar and the amount of material students can reasonably be expected to learn in their first (and perhaps only) statistics course, while bearing in mind that different instructors will naturally wish to stress different aspects of the subject. Thus, the textbook covers the full gamut of the usual statistics, with each chapter broken into subsections so that instructors may choose the particular statistics they wish to include.
- Flow charts. Flow charts that help students select appropriate statistics are provided in the textbook. The flow charts depict the selection process in detailed form at the beginning of each chapter.
- Learning objectives. Learning objectives are stated at the beginning of each chapter. These are intended to serve as "study guides" and to help students identify and focus on the most important material.
- Review of mathematical skills. A comprehensive review of all of the mathematical skills that will be used in this textbook (Prologue) is included. Students who are inexperienced or out of practice with mathematics may want to study this review early in the course and/or refer to it as needed. A self-test is included so that students may check their level of preparation for the course.
- Statistical techniques and end-of-chapter problems are explicitly linked. After a technique is introduced, students are directed to specific problems for practice and review. The "how-to-do-it" aspects of calculation are reinforced immediately and clearly.
- End-of-chapter problems are organized progressively. Simpler problems with small data sets are presented first. Often, explicit instructions or hints accompany the first several problems in a set. The problems gradually become more challenging and require more decision making by the student (e.g., choosing the most appropriate statistic for a certain situation). Thus, each problem set develops problem-solving abilities gradually and progressively.

Computer applications. To help students take advantage of the power of the computer, this textbook integrates Version 24 of IBM SPSS Statistics for Windows, though Versions 10 through 23 are completely compatible with the textbook. IBM SPSS Statistics, colloquially referred to here as "SPSS," is a leading, state-of-the-art computerized statistical package. Alternatively, students can download a full working evaluation copy of SPSS. This free 14-day trialware can be downloaded after registering at the IBM SPSS website.

The last part of each chapter, labelled "You Are the Researcher," offers demonstrations and exercises in using SPSS to produce the statistics presented in the chapter. All SPSS outputs from the demonstrations are shown in the textbook and are unedited. Students choosing to replicate the demonstration will see the same output on their computer screen. SPSS exercises are provided below the demonstrations and give students the opportunity to actually use SPSS to compute statistics. Appendix E provides additional SPSS exercises, and Appendix F gives a general introduction to SPSS.

Overall, detailed instructions and demonstrations on how to use SPSS for each exercise are provided in this textbook. SPSS can be taught at a basic level from this textbook without the aid of an SPSS supplementary text.

A manual for Stata (Version 15 for Windows), another commonly used statistical software package, is also available for the textbook. Like SPSS, Stata provides a variety of tools for statistical analyses that are easy to learn and use. The manual can be downloaded for free from the textbook's website. The manual is accompanied by two databases, allowing students to practise their statistical skills in a Stata computing environment.

Real data. Four databases for SPSS application give students a wide range of opportunities to practise their statistical skills on "real-life" data: (1) a shortened version of the 2013 Canadian General Social Survey (labelled 2013 GSS Shortened.sav); (2) a shortened version of the 2012 Canadian Community Health Survey (2012 CCHS Shortened. sav); (3) the full version of the 2013 Canadian General Social Survey (2013 GSS Full.sav); and (4) the full version of the 2012 Canadian Community Health Survey (2012 CCHS Full.sav). The databases are described in Appendix G and are available in SPSS format (i.e., the ".sav" file format) on the website for the textbook.

#### **KEY CHANGES** IN THE FOURTH CANADIAN EDITION

This edition of the textbook remains focused on developing the skills necessary for students to become statistically literate. It is written for students with varying levels of mathematical proficiency, providing an accessible yet comprehensive examination of statistics, and continues to expose students to statistics from a uniquely Canadian perspective. With this in mind, various changes were made in the fourth edition. The following are the most important changes:

- The textbook has been reorganized to better integrate the discussion of hypothesis tests and measures of association.
- New statistics and methods—index of qualitative variation, five-number summary, measures of central tendency/dispersion for grouped data, and eta-squared—have been added.
- The "Reading Statistics" inserts have been updated. Likewise, data cited in the text have been updated.
- The textbook, at the time of printing, uses the latest version of IBM SPSS Statistics for Windows (Version 24), along with the new 2013 Canadian General Social Survey data set.

INSTRUCTOR



The Nelson Education Teaching Advantage (NETA) program delivers research-based instructor resources that promote student engagement and higher-order thinking to enable the success of Canadian students and educators. Visit Nelson Education's Inspired Instruction website at nelson.com /inspired/ to find out more about NETA.

The following instructor resources have been created for Statistics: A Tool for Social Research, Fourth Canadian Edition. Access these ultimate tools for customizing lectures and presentations at nelson.com/instructor.

#### **NETA TEST BANK**



This resource was written by Markus Schafer of the University of Toronto. It includes over 500 multiple-choice questions written according to NETA guidelines for effective construction and development of higher-order questions. Also included are more than 40 problems.

The NETA test bank is available in a new, cloud-based platform. **Nelson Testing Powered by Cognero**<sup>®</sup> is a secure online testing system that allows instructors to author, edit, and manage test bank content from anywhere Internet access is available. No special installations or downloads are needed, and the desktop-inspired interface, with its dropdown menus and familiar, intuitive tools, allows instructors to create and manage tests with ease. Multiple test versions can be created in an instant, and content can be imported or exported into other systems. Tests can be delivered from a learning management system, the classroom, or wherever an instructor chooses. Nelson Testing Powered by Cognero for Statistics: A Tool for Social Research, Fourth Canadian Edition, can be accessed through nelson.com/instructor.

#### **NETA POWERPOINT**

Microsoft® PowerPoint® lecture slides for every chapter have been created by Irene Boeckmann University of Toronto. There is an average of 25 slides per chapter, many featuring key figures, and tables, from Statistics: A Tool for Social Research, Fourth Canadian Edition. NETA principles of clear design and engaging content have been incorporated throughout, making it simple for instructors to customize the deck of their courses.

#### **IMAGE LIBRARY**

This resource consists of digital copies of figures and short tables, used in the book. Instructors may use these jpegs to customize the NETA PowerPoint or create their own PowerPoint presentations. An Image Library Key describes the images and lists the codes under which the jpegs are saved.

#### **INSTRUCTOR'S MANUAL**

The Instructor's Manual to accompany Statistics: A Tool for Social Research, Fourth Canadian Edition, contains answers to even-numbered end-of-chapter problems and detailed answers to selected textbook problems. It also provides a "What can I do in class?" feature that includes 2–3 lesson ideas per chapter, and a "What can I do online?" feature, which includes 2-3 suggestions per chapter for engaging and instructing students using Web resources.

#### **APLIA**



**Aplia**<sup>TM</sup> is a Cengage Learning online homework system dedicated to improving learning by increasing student effort and engagement. Aplia makes it easy for instructors to assign frequent online homework assignments. Aplia provides students with prompt and detailed feedback to help them learn as they work through the questions, and features interactive tutorials to fully engage them in learning course concepts. Automatic grading and powerful assessment tools give instructors real-time reports of student progress, participation, and performance, and while Aplia's easy-to-use course to management features let instructors flexibly administer course announcements and materials online. With Aplia, students will show up to class fully engaged and prepared, and instructors will have more time to do what they do best . . . teach.

#### **STUDENT** ANCILLARIES



Founded in 2000 by economist and Stanford professor Paul Romer, **Aplia**™ is an educational technology company dedicated to improving learning by increasing student effort and engagement. Currently, Aplia products have been used by more than a million students at over 1,300 institutions. Aplia offers a way for you to stay on top of your coursework with regularly scheduled homework assignments that increase your time on task and give you prompt feedback. Interactive tools and additional content are provided to further increase your engagement and understanding. See Aplia.com for more information. If **Aplia** isn't bundled with your copy of *Statistics*:

A Tool for Social Research, Fourth Canadian Edition, you can purchase access separately at NELSONbrain.com. Be better prepared for class with Aplia! The **Aplia** course for *Statistic: A Tool for Social Research*, Fourth Canadian Edition, was prepared by text author Steven Prus.



The **Student Companion Site** for *Statistics* at nelson.com/student is a comprehensive, resource-rich location for students to find pertinent information. As well as SPSS data sets used for the end-of-chapter SPSS demonstrations and exercises in the textbook, they'll find additional review and study materials, including online chapters, tables of frequently used formulas and random numbers, and more! It contains:

- Data Sets and Code Books: These data sets were written for use with Version 24 of IBM SPSS Statistics for Windows, but Versions 10 through 23 are completely compatible.
- Cumulative exercises at the end of each part provides practice in choosing, computing, and analyzing statistics. These exercises present only data sets and research questions. Students must choose appropriate statistics as part of the exercise.
- **Online Chapters:** Several online chapters to accompany the text.

#### **ACKNOWLEDGMENTS** FOR THE FOURTH CANADIAN EDITION

I would like to first and foremost acknowledge and thank Dr. Teresa Abada, Western University; Dr. Alex Bierman, University of Calgary; Dr. Stephen Gyimah, Queen's University; Dr. Michelle Maroto, University of Alberta; Dr. Owen Temby, McGill University; and Dr. Michael Weinrath, University of Winnipeg, for their very helpful comments and advice in writing this textbook. The textbook and its various supplements and ancillaries have also greatly benefited from the reviews, suggestions, and guidance provided by many others. I am grateful to Catherine Gelinas, who worked diligently as technical reviewer on the textbook and solutions.

Along with others who prefer to remain anonymous, they include:

Edward Akmetshin, University of British Columbia Liqun Cao, University of Ontario Institute of Technology Daniel Cohn, York University Lesley Frank, Acadia University John Jayachandran, Concordia University Zhiqiu Lin, Carleton University Amir Mostaghim, University of Ontario Institute of Technology Patricia Pakvis, King's University College Jason Roy, Wilfrid Laurier University Markus Schafer, University of Toronto Gerry Veenstra, University of British Columbia

I wish to express a special thank you of gratitude to Riva Lieflander. This edition received extraordinary authorship from Riva. She worked on many of the key changes and additions in the fourth edition. Riva's dedication and insight have enriched and led to the successful completion of this textbook.

Maria Chu at Nelson Education provided the utmost effort and dedication to this project, and I am very thankful for her support. Finally, I would like to thank my wonderful children, Laura, Mandy, and Michael, and wife Kristen Presta. I dedicate this textbook to them.

We lastly acknowledge and are grateful to the Literary Executor of the late Sir Ronald A. Fisher, F.R.S., to Dr. Frank Yates, F.R.S., and to Longman Group Ltd., London, for permission to reprint Appendixes B, C, and D from their book Statistical Tables for Biological, Agricultural and Medical Research (6th ed., 1974). We are also grateful to Statistics Canada, for providing the General Social Survey and Canadian Community Health Survey data sets used in this textbook.

Steven G. Prus

## **Prologue: Basic Mathematics Review**

You will probably be relieved to hear that first courses in statistics are not particularly mathematical and do not stress computation per se. While you will encounter many numbers to work with and numerous formulas to use, the major emphasis will be on understanding the role of statistics in research and the logic by which we attempt to answer research questions empirically. You will also find that, at least in this textbook, the example problems and many of the homework problems have been intentionally simplified so that the computations will not unduly distract you from the task of understanding the statistics themselves.

On the other hand, you may regret to learn that there is, inevitably, some arithmetic that you simply cannot avoid if you want to master this material. It is likely that some of you haven't had any math in a long time, others have convinced themselves that they just cannot do math under any circumstances, and still others are just rusty and out of practice. However, you will find that even the most complex and intimidating operations and formulas can be broken down into simple steps. For those of you who have forgotten how to cope with some of these steps or are unfamiliar with these operations, this section is designed to ease you into the skills you will need to do all of the computation in this textbook.

#### CALCULATORS AND COMPUTERS

A calculator is a virtual necessity for this textbook. While you could do all the arithmetic by hand, a calculator will save you time and effort. Furthermore, calculator applications are conveniently available on most smartphones. While standard functions of addition, subtraction, multiplication, division, and square root are all you really need, you may want the learn additional features on your calculator including the memory and some preprogrammed functions.

Along the same lines, many of you probably have access to computers and statistical packages. If so, take the time now to learn how to use them, because they will eventually save you time and effort. This text-book includes a guide to a statistical package called IBM SPSS Statistics, but many other programs are available that will help you accomplish the goals of saving time and avoiding drudgery while generating precise and accurate results.

In summary, you should find a way at the beginning of this course—with a calculator, a statistical package, or both—to minimize the tedium and hassle of mere computing. This will permit you to devote maximum effort to the truly important goal of increasing your understanding of the meaning of statistics in particular and social research in general.

#### **VARIABLES** AND SYMBOLS

Statistics are a set of techniques by which we can describe, analyze, and manipulate variables. A variable is a trait that can change values from case to case or from time to time. Examples of variables include height, weight, level of education, and political party preference. The possible values associated with a given variable might be numerous (e.g., income) or relatively few (e.g., gender). We will often use symbols, usually the letter X, to refer to variables in general or to a specific variable.

Sometimes we will need to refer to a specific value or set of values of a variable. This is usually done with the aid of subscripts. So, the symbol  $X_1$  (read "X-sub-one") would refer to the first score in a set of scores,  $X_2$ ("X-sub-two") to the second score, and so forth. Also, we will use the subscript i to refer to all the scores in a set. Thus, the symbol  $X_i$  (read "X-sub-eye") refers to all of the scores associated with a given variable (e.g., the test grades of a particular class).

#### **OPERATIONS**

You are all familiar with the four basic mathematical operations of addition, subtraction, multiplication, and division and the standard symbols (+, -, ×, ÷) used to denote them. Some of you may not be aware, however, that the latter two operations can be symbolized in a variety of ways. For example, the operation of multiplying some number a by some number bmay be symbolized in (at least) six different ways:

$$a \times b$$

$$a \cdot b$$

$$a * b$$

$$ab$$

$$a(b)$$

$$(a)(b)$$

In this textbook, we will commonly use the "adjacent symbols" format (i.e., ab), the conventional times sign (X), or adjacent parentheses to indicate multiplication. On most calculators and computers, the asterisk (\*) is the symbol for multiplication.

The operation of division can also be expressed in several different ways. In this textbook, we will use either of these two methods:

$$a/b$$
 or  $\frac{a}{b}$ 

Several of the formulas with which we will be working require us to find the square of a number. To do this, simply multiply the number by itself. This operation is symbolized as  $X^2$  (read "X squared"), which is the same thing as (X)(X). If X has a value of 4, then

$$X^2 = (X)(X) = (4)(4) = 16$$

or we could say that "4 squared is 16."

The square root of a number is the value that, when multiplied by itself, results in the original number. So the square root of 16 is 4 because (4)(4) is 16. The operation of finding the square root of a number is symbolized as

$$\sqrt{X}$$

Be sure you have access to a calculator with a built-in square root function.

A final operation with which you should be familiar is summation, or the addition of the scores associated with a particular variable. When a formula requires the addition of a series of scores, this operation is usually symbolized as  $\sum X_i$ . The symbol  $\sum$  is the uppercase Greek letter sigma and stands for "the summation of." So the combination of symbols  $\sum X_i$  means "the summation of all the scores" and directs us to add the value of all the scores for that variable. If four people had family sizes of 2, 4, 5, and 7, then the summation of these four scores for this variable could be symbolized as

$$\sum X_i = 2 + 4 + 5 + 7 = 18$$

The symbol  $\Sigma$  is an operator, just like the + or  $\times$  signs. It directs us to add all of the scores on the variable indicated by the *X* symbol.

There are two other common uses of the summation sign, and unfortunately, the symbols denoting these uses are not, at first glance, sharply different from each other or from the symbol used above. A little practice and some careful attention to these various meanings should minimize the confusion. The first set of symbols is  $\sum X_i^2$ , which means "the sum of the squared scores." This quantity is found by first squaring each of the scores and then adding the squared scores together. A second common set of symbols will be  $(\sum X_i)^2$ , which means "the sum of the scores, squared." This quantity is found by first summing the scores and then squaring the total.

These distinctions might be confusing at first, so let's see if an example helps clarify the situation. Suppose we had a set of three scores: 10, 12, and 13. So,

$$X_i = 10, 12, 13$$

The sum of these scores would be indicated as

$$\sum X_i = 10 + 12 + 13 = 35$$

The sum of the squared scores would be

$$\sum X_i^2 = (10)^2 + (12)^2 + (13)^2 = 100 + 144 + 169 = 413$$

Take careful note of the order of operations here. First, the scores are squared one at a time and then the squared scores are added. This is a completely different operation from squaring the sum of the scores:

$$(\sum X_i)^2 = (10 + 12 + 13)^2 = (35)^2 = 1.225$$

To find this quantity, first the scores are summed and then the total of all the scores is squared. The value of the sum of the scores squared (1,225) is not the same as the value of the sum of the squared scores (413). In summary, the operations associated with each set of symbols are as follows:

Symbols	Operations
$\sum X_i$	Add the scores
$\sum X_i^2$	First square the scores and then add the squared scores
$(\Sigma X_i)^2$	First add the scores and then square the total

#### OPERATIONS WITH NEGATIVE NUMBERS

A number can be either positive (if it is preceded by a + sign or by no sign at all) or negative (if it is preceded by a - sign). Positive numbers are greater than zero, and negative numbers are less than zero. It is very important to keep track of signs because they will affect the outcome of virtually every mathematical operation. This section will briefly summarize the relevant rules for dealing with negative numbers. First, adding a negative number is the same as subtraction. For example,

$$3 + (-1) = 3 - 1 = 2$$

Second, subtraction changes the sign of a negative number:

$$3 - (-1) = 3 + 1 = 4$$

Note the importance of keeping track of signs here. If you neglected to change the sign of the negative number in the second expression, you would arrive at the wrong answer.

For multiplication and division, there are various combinations of negative and positive numbers you should be aware of. For purposes of this textbook, you will rarely have to multiply or divide more than two numbers at a time, and we will confine our attention to this situation. Ignoring the case of all positive numbers, this leaves several possible combinations. A negative number multiplied by a positive number results in a negative value:

$$(-3)(4) = -12$$

or

$$(3)(-4) = -12$$

A negative number multiplied by a negative number is always positive:

$$(-3)(-4) = 12$$

Division follows the same patterns. If there is a single negative number in the calculations, the answer will be negative. If both numbers are negative, the answer will be positive. So,

$$\frac{-4}{2} = -2$$

and

$$\frac{4}{-2} = -2$$

but

$$\frac{-4}{-2} = 2$$

Note that negative numbers do not have square roots, because multiplying a number by itself cannot result in a negative value. Squaring a negative number always results in a positive value (see the multiplication rules above).

#### ACCURACY AND ROUNDING OFF

A possible source of confusion in computation involves the issues of accuracy and rounding off. People work at different levels of accuracy and precision and, for this reason alone, may arrive at different answers to problems. This is important, because if you work at one level of precision and the textbook (or your instructor or your study partner) works at another, the resulting solutions will be at least slightly different. You may sometimes think you've gotten the wrong answer when all you've really done is round off at a different place in the calculations or in a different way.

There are two issues here: when to round off and how to round off. In this textbook, we have followed the convention of working with as much accuracy as our calculator or statistics package will allow and then rounding off to two places of accuracy (two places beyond the decimal point) at the very end. If a set of calculations is lengthy and requires the reporting of intermediate sums or subtotals, we will round the subtotals off to two places also.

In terms of how to round off, begin by looking at the digit immediately to the right of the last digit you want to retain. If you want to round off to 100ths (two places beyond the decimal point), look at the digit in the 1000ths place (three places beyond the decimal point). If that digit is greater than 5, round up. For example, 23.346 would round off to 23.35. If the digit to the right is less than 5, round down. So, 23.343 would become 23.34. If the digit to the right is 5, round up if the digit immediately to the left is even and round down if the digit is odd. So, 23.345 would become 23.35 and 23.355 would round to 23.35.

Let's look at some more examples of how to follow the rounding rules stated above. If you are calculating the mean value of a set of test scores and your calculator shows a final value of 83.459067, and you want to round off to two places beyond the decimal point, look at the digit three places beyond the decimal point. In this case the value is 9 (greater than 5), so we would round the second digit beyond the decimal point up and report the mean as 83.46. If the value had been 83.453067, we would have reported our final answer as 83.45. A value of 83.455067 would round to 83.45, and a value of 83.445067 would be 83.45.

FORMULAS, COMPLEX OPERATIONS, AND THE ORDER OF **OPERATIONS** 

A mathematical formula is a set of directions, stated in general symbols, for calculating a particular statistic. To "solve a formula" means that you must replace the symbols with the proper values and then manipulate the values through a series of calculations. Even the most complex formula can be rendered manageable if it is broken down into smaller steps. Working through these steps requires some knowledge of general procedure and the rules of precedence of mathematical operations. This is because the order in which you perform calculations may affect your final answer. Consider the following expression:

$$2 + 3(4)$$

Note that if you do the addition first, you will evaluate the expression as

$$5(4) = 20$$

but if you do the multiplication first, the expression becomes

$$2 + 12 = 14$$

Obviously, it is crucial to complete the steps of a calculation in the correct order.

The basic rules of precedence are to find all squares and square roots first, then do all multiplication and division, and finally complete all addition and subtraction. So the following expression:

$$8 + 2 \times 2^2/2$$

would be evaluated as

$$8 + 2 \times \frac{4}{2} = 8 + \frac{8}{2} = 8 + 4 = 12$$

The rules of precedence may be overridden when an expression contains parentheses. Solve all expressions within parentheses before applying the rules stated above. For most of the complex formulas in this textbook, the order of calculations will be controlled by the parentheses. Consider the following expression:

$$(8 + 2) - 4(3)^2/(8 - 6)$$

Resolving the parenthetical expressions first, we would have

$$(10) - 4 \times 9/(2) = 10 - 36/2 = 10 - 18 = -8$$

Without the parentheses, the same expression would be evaluated as

$$8 + 2 - 4 \times 3^{2}/8 - 6$$

$$= 8 + 2 - 4 \times 9/8 - 6$$

$$= 8 + 2 - 36/8 - 6$$

$$= 8 + 2 - 4.5 - 6$$

$$= 10 - 10.5$$

$$= -0.5$$

A final operation you will encounter in some formulas in this textbook involves denominators of fractions that themselves contain fractions. In this situation, solve the fraction in the denominator first and then complete the division. For example,

$$\frac{15-9}{6/2}$$

would become

$$\frac{15-9}{6/2} = \frac{6}{3} = 2$$

When you are confronted with complex expressions such as these, don't be intimidated. If you're patient with yourself and work through them step by step, beginning with the parenthetical expression, even the most imposing formulas can be managed.

#### **EXERCISES**

You can use the problems below as a "self-test" on the material presented in this review. If you can handle these problems, you're ready to do all of the arithmetic in this textbook. If you have difficulty with any of these problems, please review the appropriate section of this prologue. You might also want to use this section as an opportunity to become more familiar with your calculator. Answers are given on the next page, along with some commentary and some reminders.

1. Complete each of the following:

**a.** 
$$17 \times 3 =$$

**b.** 
$$17(3) =$$

**c.** 
$$(17)(3) =$$

**d.** 
$$17/3 =$$

**e.** 
$$(42)^2 =$$

**f.** 
$$\sqrt{113} =$$

**2.** For the set of scores  $(X_i)$  of 50, 55, 60, 65, and 70, evaluate each of the expressions below:

$$\sum X_i =$$

$$\sum X_i^2 =$$

$$(\sum X_i)^2 =$$

**3.** Complete each of the following:

**a.** 
$$17 + (-3) + (4) + (-2) =$$

**b.** 
$$15 - 3 - (-5) + 2 =$$

**c.** 
$$(-27)(54) =$$

**d.** 
$$(113)(-2) =$$

**e.** 
$$(-14)(-100) =$$

**f.** 
$$-34/-2 =$$

**g.** 
$$322/-11 =$$

- 4. Round off each of the following to two places beyond the decimal point:
  - **a.** 17.17532
  - **b.** 43.119
  - **c.** 1,076.77337
  - **d.** 32.4651152301
  - **e.** 32.4751152301
- **5.** Evaluate each of the following:

**a.** 
$$(3 + 7)/10 =$$

**b.** 
$$3 + 7/10 =$$

**c.** 
$$\frac{(4-3)+(7+2)(3)}{(4+5)(10)} =$$

#### ANSWERS TO EXERCISES

- **1. a.** 51 **b.** 51 **c.** 51 (The obvious purpose of these first three problems is to remind you that there are several different ways of expressing multiplication.)
  - **d.** 5.67 (Note the rounding off.)
- **e.** 1,764
- **f.** 10.63
- **2.** The first expression translates to "the sum of the scores," so this operation would be

$$\sum X_i = 50 + 55 + 60 + 65 + 70 = 300$$

The second expression is the "sum of the squared scores." So

$$\sum X_i^2 = (50)^2 + (55)^2 + (60)^2 + (65)^2 + (70)^2$$

$$\Sigma X_i^2 = 2,500 + 3,025 + 3,600 + 4,225 + 4,900$$

$$\sum X_i^2 = 18,250$$

The third expression is "the sum of the scores, squared":

$$(\sum X_i)^2 = (50 + 55 + 60 + 65 + 70)^2$$

$$(\sum X_i)^2 = (300)^2$$

$$(\sum X_i)^2 = 90,000$$

Remember that  $\sum X_i^2$  and  $(\sum X_i)^2$  are two completely different expressions with very different values.

- **3. a.** 16 **b.** 19 (Remember to change the sign of -5.) **c.** -1,458
  - **d.** −226
- **e.** 1,400 **f.** 17 **g.** −29.27
- **4. a.** 17.17 **b.** 43.12
- **c.** 1,076.77
- **d.** 32.47 **e.** 32.47
- **b.** 3.7 (Note again the importance of parentheses.)
- **c.** 0.31